ÉRETTSÉGI VIZSGA • 2010. október 19.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

NEMZETI ERŐFORRÁS MINISZTÉRIUM

Instructions to examiners

Formal requirements:

1. Mark the paper in **ink, different in colour** from the one used by the candidate. Indicate the errors, incomplete solutions, etc. in the conventional way.
2. The first one of the grey rectangles under each problem shows the maximum attainable score on that problem. The **points** given by the examiner are to be entered **in the rectangle** next to that.
3. **If the solution is perfect**, it is enough to enter the maximum scores in the appropriate rectangles.
4. If the solution is incomplete or incorrect, please indicate the individual **partial scores** in the body of the paper, too.

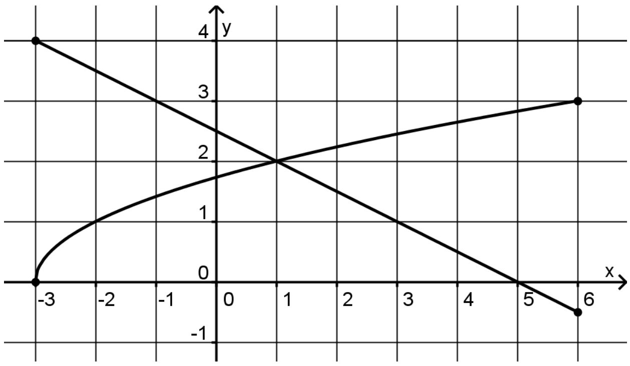
Assessment of content:

1. The markscheme may contain more than one solution for some of the problems. If the **solution by the candidate is different**, allocate the points by identifying the parts of the solution equivalent to those of the one given in the markscheme.
2. The subtotals in the markscheme can be **further divided**, but the scores awarded should always be whole numbers.
3. If it is clear that the reasoning and the final answer are both correct, you may award the maximum score even if the solution is **less detailed** than the one in the markscheme.
4. If there is a **calculation error** or inaccuracy in the solution, only take off the points for that part where the error occurs. If the reasoning remains correct and the error is carried forward without changing the nature of the task, the points for the rest of the solution should be awarded.
5. In the case of a **principal error**, no points should be awarded at all for that section of the solution, not even for formally correct steps. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the wrong information based on the principal error is carried forward to the next section or to the next part of the problem and is used correctly there, the maximum score is due for the next part, provided that the error has not changed the nature of the task.
6. Where the markscheme shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.
7. If there are more than one different approaches to a problem, **assess only the one indicated by the candidate**.
8. **Do not give extra points** (i.e. more than the maximum score due for the problem or part of problem).
9. **Do not take off points** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **Assess only four out of the five problems in part II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted in their total score. Should there be a solution to that problem, it does not need to be marked. If it is not clear which problem the candidate does not want to be assessed, assume automatically that it is the last one in the question paper, and do not assess that problem.

I.

1. a)		
By expanding the cubes, $(x^3-3x^2+3x-1)-(x^3+3x^2+3x+1)>-8$	2 points	
Hence $x^2<1$,	1 point	
and the set of solutions is the interval $]-1;1[$.	1 point	
Total:	**4 points**	

1. b)



Correct graph of function *f*	2 points	
Correct graph of function *g*	1 point	
The coordinates of the intersection are $(1;2)$.	1 point	*Only due if there is a proof by calculation.*
Total:	**4 points**	

1. c) Solution 1.		
The original inequality is equivalent to $\sqrt{x+3}\le -0.5x+2.5$.	1 point	
The expression on the left-hand side is non-negative.	1 point	
The expression on the right-hand side is negative if x is greater than 5.	1 point	
The solution of the inequality can be read from the graph obtained in part b) for the functions *f* and *g* on the interval $[-3;6]$.	1 point	
The set of solutions is the interval $[-3;1]$.	2 points	
Total:	**6 points**	

1. c) **Solution 2.**		
The original inequality is equivalent to $\sqrt{x+3} \le -0.5x + 2.5$, where $x \ge -3$.	1 point	
There is no solution in the set $[5\ ;\ \infty[$ since in that case the left-hand side is positive and the right-hand side is negative or 0.	1 point	
For $x \in [-3;5[$, both sides of the inequality are non-negative. Squaring is an equivalent transformation on this set:	1 point	
$x + 3 \le 0.25x^2 - 2.5x + 6.25$.	1 point	
$0 \le x^2 - 14x + 13$. The set of solutions on **R** is $[-\infty;1] \cup [13\,;\infty[$.	1 point	
Thus the solution on the set $[-3;5[$ is the interval $[-3\,;1\,]$.	1 point	
Total:	**6 points**	

2. a)		
The digit with the largest place value must be an 8.	1 point	*The point is also due if this idea is only reflected by the solution.*
Each of the digits in the nine other places may be of two kinds: 0 or 8.	1 point	
That is 2^9 (= 512) ten-digit numbers.	1 point	
Total:	**3 points**	

2. b)		
A number is divisible by 45 if and only if it is divisible by 9 and 5.	2 points	
Since the number in question is divisible by 5, it must end in 0.	1 point	
A (positive whole) number is divisible by 9 if and only if the sum of its digits is divisible by 9.	1 point	
In the case of a number containing digits of 0 and 8 only, that takes at least nine digits of 8.	1 point	
The smallest (positive) multiple has exactly nine digits of 8,	1 point	
thus the number in question is 8 888 888 880.	1 point	
Total:	**7 points**	

3. a)		
The area of the base is $T_{ABCD} = 12 \cdot 6 = 72$ cm².	1 point	
Let *M* be the midpoint of edge *AB*, and let *N* be the midpoint of edge *CD*. Triangle *APB* is isosceles, *PM* is perpendicular to line segment *AB*. Triangle *MNP* has a right angle at vertex *N*, since line segment *PN* is perpendicular to plane *ABCD* and thus it is also perpendicular to every line of that plane.	1 point	*The 1 point is also due if this reasoning is less detailed or it is only reflected by the steps of the solution.*
PM = 10 (cm) (the legs are 6 and 8).	1 point	
The area of triangle *ABP* is $T_{ABP} = \frac{AB \cdot PM}{2} = \frac{12 \cdot 10}{2} = 60$ (cm²).	1 point	
Triangle *DCP* is isosceles, its area is $T_{DCP} = \frac{DC \cdot PN}{2} = \frac{12 \cdot 8}{2} = 48$ (cm²).	1 point	
DP=*PC*=10 (cm) (e.g. from right-angled triangle *PCG*, in which the legs are 8 and 6).	1 point	
Lateral faces *PBC* and *PAD* are congruent triangles (sides are pairwise equal),	1 point	*These points may also be awarded as follows: Edge BC is perpendicular to plane CDHG and thus to all lines in it, too: 1 point. So triangle BCP is right-angled (at vertex C): 1 point.*
and by the equality of corresponding sides, the two triangles are also congruent to (e.g.) triangle *PBM* that is right-angled (at vertex *M*).	1 point	
$T_{PBC} = \frac{6 \cdot 10}{2} = 30$ (cm²).	1 point	
The surface area of the pyramid is $(72 + 60 + 48 + 2 \cdot 30 =)$ 240 cm².	1 point	
Total:	**10 points**	

3. b) Solution 1.		
The plane of face *ABP* is the plane of the diagonal section *ABGH* of the cuboid,	1 point	
thus the angle of the two planes equals $\sphericalangle HAD$.	1 point	
$\tan \sphericalangle HAD = \frac{HD}{AD} = \frac{8}{6} = \frac{4}{3}$, and hence $\sphericalangle HAD \approx 53.1°$	1 point	*Accept any correct approximate value.*
Total:	**3 points**	

3. b) **Solution 2.**		
Line segments *MN* and *PM* are both perpendicular to edge *AB*, therefore the angle in question is $\sphericalangle PMN$.	1 point	
Triangle *PMN* has a right angle at *N*,	1 point	
so $\tan \sphericalangle PMN = \frac{PN}{MN} = \frac{8}{6} = \frac{4}{3}$, and hence $\sphericalangle PMN \approx 53.1°$.	1 point	*Accept any correct approximate value.*
Total:	**3 points**	

4. a)		
The number of boys is calculated from the data in the columns of the table:	1 point	*The point is also due if this idea is only revealed by the calculation.*
(103 + 58 + 15 + 3 + 3 + 0) + 2·(61 + 11 + 3 + 3·1) + + 3·16 + 4·9 + 5·4 =	1 point	
= 442 boys altogether in the families examined.	1 point	
Total:	**3 points**	

4. b)		
The number of girls is obtained from the rows, not counting the number of families with no children or with a single child (160, 103 and 121 families). There are no girls in 61+8+5=74 families.	1 point	
1 girl in 58+11+4+1+1=75 families.	1 point	
2 girls in 54+15+3+2+2+2=78 families.	1 point	
3 girls in 9+3+1+1+1+1=14 families. 4 girls in 6+3+1+1+1+1=12 families. 5 girls in 1+1= 2 families.	1 point	*This point is also due if the candidate only states that it is not possible to get a sum larger than 78 any more, but does not calculate the actual sums.*
The most frequently occurring number of girls in families with at least two children is 2.	1 point	
Total:	**5 points**	

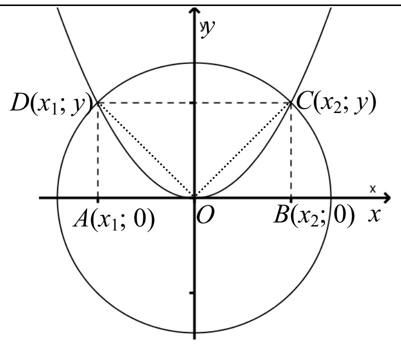
4. c)

number of children in a family	4	5	6	7	8	9	10
frequency	21	8	5	4	2	0	0

Correct interpretation of frequency.	1 point	*The point is due if this is reflected by the solution.*
At least 4 correct frequencies in the table.	1 point	
All frequencies correct.	1 point	
The number of supported families is 40.	1 point	*These points are also awarded if the calculation is correct and accurate but uses wrong data carried forward.*
The number of children supported is $21\cdot4+8\cdot5+5\cdot6+4\cdot7+2\cdot8=$	1 point	
= 198.	1 point	
Total:	**6 points**	

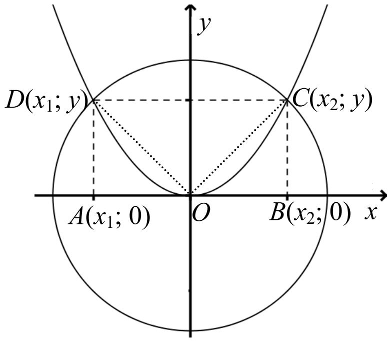
II.

5. Solution 1.



The centre of the circle $x^2 + y^2 = 8$, and the vertex of the parabola are both at the origin (O).	2 points	*These 2 points are also due if there is an appropriate diagram.*
Finding the intersections: $\left.\begin{array}{l} 2y = x^2 \\ x^2 + y^2 = 8 \end{array}\right\}$ $y^2 + 2y - 8 = 0$ $y_1 = 2 \quad y_2 = -4$.	2 points	
Only $y = 2$ satisfies the conditions.	1 point	
The abscissas of the intersections are $x_1 = -2$ and $x_2 = 2$.	1 point	
The central angle of the circular segment intercepted by chord CD is $\frac{\pi}{2}$ radians (=90°),	1 point	
since OD and OC are diagonals of squares.	1 point	*The 1 point is also due if this idea is reflected by the solution.*
So the area of the circular segment is $T_{\text{circular}} = \frac{1}{2}r^2(\hat{\alpha} - \sin\alpha) =$ $= \frac{1}{2} \cdot 8 \cdot \left(\frac{\pi}{2} - \sin\frac{\pi}{2}\right) = 2\pi - 4$.	2 points	$2\pi - 4 \approx 2.283$
The area of the parabolic segment intercepted by the chord CD is $T_{\text{parabolic}} = T_{ABCD} - \int_{x_1}^{x_2} \frac{x^2}{2}dx = 4 \cdot 2 - \int_{-2}^{2} \frac{x^2}{2}dx =$ $= 8 - \left[\frac{x^3}{6}\right]_{-2}^{2} = 8 - \left[\frac{4}{3} - \left(-\frac{4}{3}\right)\right] = \frac{16}{3}$	5 points	*In awarding partial scores, consider the number of correct equalities.*
The area of the convex part is $T = T_{\text{circular}} + T_{\text{parabolic}} = 2\pi - 4 + \frac{16}{3} = 2\pi + \frac{4}{3}$ units of area.	1 point	$2\pi + \frac{4}{3} \approx 7.62$
Total:	**16 points**	

Take off 2 points from the score if an approximate value of π is used in the calculation.

5. Solution 2.		
		
The centre of the circle $x^2 + y^2 = 8$, and the vertex of the parabola are both at the origin (O).	2 points	*These 2 points are also due if there is an appropriate diagram.*
Finding the intersections: $\left.\begin{array}{l} 2y = x^2 \\ x^2 + y^2 = 8 \end{array}\right\}$ $y^2 + 2y - 8 = 0$ $y_1 = 2 \quad y_2 = -4$.	2 points	
Only $y = 2$ satisfies the conditions.	1 point	
The abscissas of the intersections are $x_1 = -2$ and $x_2 = 2$.	1 point	
The central angle of the circular segment intercepted by chord CD is $\frac{\pi}{2}$ radians (=90°),	1 point	
since OD and OC are diagonals of squares.	1 point	*The 1 point is also due if this idea is reflected by the solution.*
So the area of the circular sector is $T_{\text{sector}} = \frac{1}{4} r^2 \pi = 2\pi.$	1 point	
The area in question is obtained by adding the double of the area of the segment cut out of the parabola by chord OC to the area of the quarter circle.	2 points	
The area of the segment cut out of the parabola by chord OC is obtained by subtracting the area under the parabola over interval $[0\,;2]$ from the area of the right-angled triangle OBC.	1 point	
$T_{\text{segment}} = T_{OBC} - \int_0^2 \frac{x^2}{2} dx = 2 - \left[\frac{x^3}{6}\right]_0^2 = 2 - \frac{4}{3} = \frac{2}{3}$	3 points	*In awarding partial scores, consider the number of correct equalities.*
$T = T_{\text{sector}} + 2T_{\text{segment}} = 2\pi + 2 \cdot \frac{2}{3} = 2\pi + \frac{4}{3}$ units of area.	1 point	$2\pi + \frac{4}{3} \approx 7.62$
Total:	**16 points**	

6. a) **Solution 1.**		
The triangles in question are bounded by • two sides and a diagonal; • one side and (the lines of) two diagonals; • (the lines of) three diagonals of the pentagon *ABCDE*.	2 points	*These 2 points are awarded for the use of any kind of systematic method of counting the triangles. The 2 points are also due if no correct counting principle is formulated but a correct result is obtained.*
There are 5 triangles in which two sides are two consecutive sides of the large pentagon.	1 point	
There are $5 \cdot 4=20$ triangles in which only one side is a side of the big pentagon. (For example, the triangles on side *AB* are *ABR*, *ABS*, *ABT*, *ABD*.)	1 point	
(Since the lines of any three diagonals determine a triangle,) there are $\binom{5}{3}=10$ triangles in which all sides lie on diagonals.	1 point	
The triangles listed above are all different, so there are 35 triangles in the diagram.	1 point	
Significantly different triangles differ in their angles, too. The angles of the triangles listed above are either 36°, 36° and 108°, or 72°, 72° and 36°.	1 point	*The point is also due if this idea appears in a diagram or sketch.*
Therefore, there are two kinds of significantly different triangles in the diagram.	1 point	
Total:	**8 points**	

6. a) **Solution 2.**		
The angles of the triangles in question are either 36°, 36° and 108°, or 72°, 72° and 36°.	1 point	
Thus there are two kinds of significantly different triangles in the diagram.	1 point	
Triangles with angles of 36°, 36° and 108° occur in two sizes: the longest side is either a diagonal or a side of pentagon *ABCDE*.	1 point	
The number of such triangles is 10+5=15.	1 point	
Triangles with angles of 72°, 72° and 36° occur in three sizes: the shortest side may be a side of pentagons *ABCDE* or *PQRST*, or it may be a side of the five-pointed star polygon.	2 points	
The number of such triangles is 5+5+10=20.	1 point	
There are 35 triangles in the diagram altogether.	1 point	
Total:	**8 points**	

6. b)		
Quadrilateral *ABCQ* is a rhombus since its opposite angles are equal: 72° and 108°. If *a* denotes the side of the pentagon (and the rhombus), then $a^2 \cdot \sin 108° = 120$. $(a \approx 11.232\ \text{cm})$.	1 point	
The area of the regular pentagon is 5 times the area of the triangles formed with centre *O* (area *ABO*). The height drawn to base *a* is $m = \frac{a}{2} \cdot \tan 54°$, thus the area is $T_{ABCDE} = 5 \cdot \frac{a \cdot m}{2} = \frac{5}{4} a^2 \cdot \tan 54°$.	1 point	
$T_{ABCDE} = \frac{5}{4} \cdot \frac{120}{\sin 108°} \cdot \tan 54°$.	1 point	
$T_{ABCDE} \approx 217\ \text{cm}^2$.	1 point	
Total:	**4 points**	

6. c)		
Statement 1: true,	1 point	
since the degree of each of the 10 vertices is 4, the sum of the degrees is 40, which is the double of the number of edges.	1 point	
Statement 2: true,	1 point	
for example *ABCDEQPTA* is a circuit of eight vertices.	1 point	
Total:	**4 points**	

7. a)		
The monthly revenue from selling the cream is $x(36 - 0.03x)$ euros.	1 point	*Award no points for part a) if the candidate does not obtain a quadratic function for the revenue.*
The quadratic function $x \mapsto -0.03x^2 + 36x$ $(x \in \mathbf{R})$ has a maximum.	1 point	*The 1 point is due if this idea is reflected by the solution.*
Its zeros are 0 and 1200,	1 point	
so its maximum occurs at 600.	1 point	
This point lies in the given interval.	1 point	
Thus the maximum revenue is achieved by selling 600 kg of cream, and the maximum revenue is 10 800 euros.	1 point	
Total:	**6 points**	

7. b)		
The monthly profit equals the difference of the monthly revenue and the monthly cost. The monthly profit is given by the function $x \mapsto -0.03x^2 + 36x - \left(0.0001x^3 - 30.12x + 13000\right)$ $(100 < x < 700)$.	1 point	*The 1 point is due if this idea is reflected by the solution.*
The profit function is $x \mapsto -0.0001x^3 - 0.03x^2 + 66.12x - 13\,000$ $(100<x<700)$.	1 point	
This function is differentiable, and its derivative is the function $x \mapsto -0.0003x^2 - 0.06x + 66.12$ $(100<x<700)$.	1 point	
The equation $-0.0003x^2 - 0.06x + 66.12 = 0$ $(x^2 + 200x - 220\,400 = 0)$ has one negative ($x_1 = -580$) and one positive real root ($x_2 = 380$).	1 point	
The derivative function is positive on the interval]100; 380[,	1 point	
and negative on the interval]380; 700[,	1 point	
so the profit function strictly increases up to $x = 380$, and then strictly decreases.	1 point	
Thus the function in question has a single absolute maximum, and that occurs at 380.	1 point	
The maximum value of the function is 2306.4.	1 point	
Therefore, the greatest monthly profit is achieved by selling 380 kg of cream, and its value is 2306.4 euros.	1 point	
Total:	**10 points**	

8. a)		
Miki may have paid in two ways: 240 = 200+20+10+10 = 100+100+20+20.	2 points	
Karcsi may have paid in four ways: 240 = 200+20+10+5+5 = 200+10+10+10+10	1 point	
240 = 100+100+20+10+10 = 100+50+50+20+20.	1 point	
Total:	**4 points**	

8. b)		
Bandi may win the jackpot in three cases: (1) He wins the jackpot in the first draw, and then he wins the jackpot in the second draw again (when the same numbers are drawn twice). The probability of this event is $p \cdot p = p^2$.	1 point	
(2) He wins the jackpot in the first draw, and he does not win the jackpot in the second draw. The probability of this event is $p \cdot (1-p) = p - p^2$.	1 point	
(3) He does not win the jackpot in the first draw, and he wins the jackpot in the second draw. The probability of this event is $(1-p) \cdot p = p - p^2$.	1 point	
The probability of Bandi winning the jackpot on a certain day is the sum of these three probabilities: $2p - p^2$ (this is non-negative since $0 < p < 1$).	1 point	
Total:	**4 points**	
Allocation of points for a solution based on the complementary event: *The complementary event: he does not win the jackpot in either of the two draws:* *1 point* *The probability of this event is* $(1-p)^2$, *1 point* *that is, the probability of making the jackpot at least once is* $1-(1-p)^2 = 2p - p^2$. *2 points*		

8. c)		
There are two cases to investigate, depending on whether Bandi fills out his two tickets identically or differently.	1 point	*This 1 point is also due if this idea is only reflected by the solution.*
(1) If Bandi has two identical tickets then the probability of his winning the jackpot is *p*.	1 point	
(2) If Bandi has two different tickets then the probability of his winning the jackpot is 2*p*.	2 points	
Total:	**4 points**	

8. d)		
If Bandi has two identical tickets, the probabilities to be compared are $2p - p^2$ and p.	1 point	
Since $0 < p < 1$, it follows that $2p - p^2 - p = p(1-p) > 0$, therefore game **b)** is more favourable.	1 point	
If Bandi has two different tickets, the probabilities to be compared are $2p - p^2$ and $2p$.	1 point	
Since $p^2 > 0$, it follows that $2p - p^2 < 2p$, therefore game **c)** is more favourable.	1 point	
Total:	**4 points**	

9. a) Solution 1.

The table below illustrates the conditions: x students do not have a certificate in German, and $(10580-x)$ students do.

	no certificate in German (x students)	has certificate in German $(10580-x)$ students
no certificate in English	has neither German nor English	has German but no English
has certificate in English	has English but no German	has both German and English

Correct interpretation of the problem (complementary sets).	1 point	
According to the condition of the problem, 70% of the x students, that is $0.7x$ students have no certificates in either German or English;	1 point	
and 30% of $(10580-x)$ students, that is $0.3\cdot(10580-x)$ students have a certificate in German but have no certificate in English.	1 point	
Thus the number of students with no certificate in English is $0.7x+0.3\cdot(10580-x)=$	1 point	
$=3174+0.4x$.	1 point	
According to the condition of the problem, 60% of these $(3174+0.4x)$ students, that is $0.6\cdot(3174+0.4x)$ students have no certificate in either language. Therefore	1 point	
$0.7x=0.6\cdot(3174+0.4x)$.	1 point	
Hence $x=4140$.	2 points	
The number of those with certificates in German is $(10580-x)=6440$ students.	1 point	
The number of those with no certificate in English is $3174+0.4x=4830$.	1 point	
Hence the number of students with certificates in English is 10 580–4 830=5750.	1 point	
Total:	**12 points**	

9. a) Solution 2.

The table below illustrates the conditions: n students do not have a certificate in either German or English.

	no certificate in German	has certificate in German
no certificate in English	has neither German nor English (n students)	has German but no English
has certificate in English	has English but no German	has both German and English

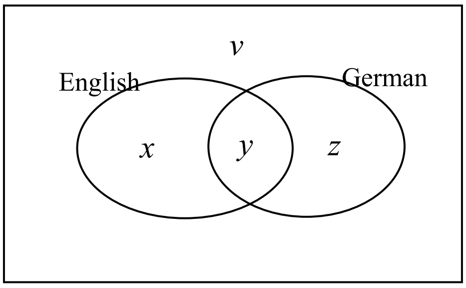
Correct interpretation of the problem (complementary sets).	1 point	
According to the condition of the problem, $\frac{100}{70}$ of the n students, that is $\frac{10}{7}n$ students have no certificate in German, and $\left(10\,580-\frac{10}{7}n\right)$ students do have certificates in German.	1 point	
30% of the latter students, that is $0.3\cdot\left(10\,580-\frac{10}{7}n\right)$ students have certificates in German but have none in English.	1 point	
Thus the number of those with no certificate in English is $n+0.3\cdot\left(10\,580-\frac{10}{7}n\right)=$	1 point	
$=3\,174+\frac{4}{7}n$ students.	1 point	
According to the condition of the problem, 60% of the $\left(3174+\frac{4}{7}n\right)$ students, that is $0.6\cdot(3174+\frac{4}{7}n)$ students have no certificates in either language. Therefore	1 point	
$n=0.6\cdot(3\,174+\frac{4}{7}n)$.	1 point	
Hence $n=2898$.	2 points	
The number of those having a German certificate is $\left(10\,580-\frac{10}{7}n\right)=6440$ students.	1 point	
The number of those with no certificate in English is $3174+\frac{4}{7}n=4830$.	1 point	
Hence the number of students with certificates in English is 10 580–4830=5750.	1 point	
Total:	**12 points**	

The set diagram completed with the appropriate numbers of elements:

	no certificate in German (4140 students)	has certificate in German (6440 students)
no certificate in English (4830 students)	has neither German nor English (2898 students)	has German but no English (1932 students)
has certificate in English (5750 students)	has English but no German (1242 students)	has both German and English (4508 students)

9. b)		
30% of the 6440 students with certificates in German (that is 1932 students) have no certificates in English.	1 point	
That is, 70% of those with certificates in German also have certificates in English. Their number is 4508.	1 point	
$\frac{4\,508}{10\,580} = 0.426$.	1 point	
42.6% of the students have certificates in both German and English.	1 point	
Total:	**4 points**	

Allocation of points in the case of a solution using four unknowns:



Set diagram or clear definition of the unknowns. *2 points*

$0.7(x+v) = v$

$0.3(y+z) = z$

$0.6(v+z) = v$

$x + y + z + v = 10\,580$

1 point for each equation. *(4 points)*
Correct steps of rearrangement to calculate the unknowns. *3 points*
Answer: $x = 1242$, $y = 4508$, $z = 1932$, $v = 2898$.
Correct solution of the simultaneous equations. *4 points*

a) $x + y = 5750$ *(number of those with English certificates),* $y + z = 6440$ *(number of those with German certificates.)*
1 point for each correct answer. *(2 points)*

b) 42.6% of the students have certificates in both German and English.
Correct answer. *1 point*